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## Breakwater and Revetment Armor Stability

by Jeffrey A. Melby

**PURPOSE:** This Coastal and Hydraulics Engineering Technical Note (CHETN) provides improved predictive equations for coastal rubble-mound stone-armor stability. The engineering methods outlined apply to breakwaters and revetments exposed to wave loading. The stability equations are based on the concept that the maximum wave force causing armor instability is proportional to the maximum wave momentum flux near the structure toe. This concept introduces a more physics-based first principles approach to estimation of armor stability. The new equations explicitly include the effects of nearshore wave height, wave period, water depth, and storm duration as well as the characteristics of wave breaking on the structure.

**INTRODUCTION:** Armor stability on large coastal rubble mounds has historically been based on empirical equations that relate armor buoyant weight to the maximum wave drag force. The fluid velocity in the maximum drag force was computed using the shallow-water wave celerity  $C = \sqrt{gH}$  where  $g$  is the acceleration of gravity and  $H$  is the wave height near the structure. Hudson (1959) and other early researchers used this formulation to develop the stability number  $N_s = H/\Delta D_{n50}$  as the basic measuring stick for stability. Here  $\Delta = S_r - 1$  is the buoyant density of the stone,  $S_r$  is the stone specific gravity, and  $D_{n50} = (V_{n50})^{1/3}$  where  $V_{n50}$  is the median stone volume. Van der Meer (1987) also used this formulation of the stability parameter. The weakness in these methods is that the fluid force is too simplistic to be generally applicable. For shallow water depth-limited applications, the maximum wave force depends on the local water depth and wave period. Therefore these variables should be included in the derivation of the equations.

**MOMENTUM FLUX:** Melby and Hughes (2004) derived a formulation for armor stability by assuming the maximum wave force was proportional to the maximum wave momentum flux. Hughes (2003a, 2003b, 2004, 2005) introduced the wave momentum flux for coastal applications. Assuming irrotational potential flow on a locally flat bottom in water depth  $h$ , the wave-averaged and depth-integrated radiation stress is given by the integration of wave momentum flux over the wavelength, i.e.,

$$M = S_{xx} = \frac{1}{L} \int_0^L \int_{-h}^{\eta_x} (p_d + \rho_w u^2) dz dx \quad (1)$$

where  $L$  is the wavelength,  $\eta_x$  is the free surface location,  $p_d$  is the dynamic pressure,  $\rho_w$  is the fluid density,  $u$  is the velocity in the  $x$ -direction,  $x$  is the horizontal coordinate, and  $z$  is the vertical coordinate. The maximum depth-integrated wave momentum flux is given at the wave crest as:

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$$(M_F)_{\max} = \int_{-h}^{H/2} (p_d + \rho u^2) dz \quad (2)$$

Using linear wave theory values for  $u$  and  $p_d$  yields

$$\left( \frac{M_F}{\rho_w g h^2} \right)_{\max} = \frac{1}{2} \frac{H}{h} \frac{\tanh kh}{kh} + \frac{1}{8} \left( \frac{H}{h} \right)^2 \left( 1 + \frac{2kh}{\sinh 2kh} \right) \quad (3)$$

where  $k = 2\pi/L$  is the wave number. In Equation 3, the first term on the right-hand side is the dynamic pressure term while the second is the velocity term. In general, the pressure term will dominate. For example, for low steepness waves, the velocity term will only contribute 5 percent to the maximum momentum flux. For waves in shallow water at the steepness limit, the velocity term will provide the maximum contribution, roughly 30 percent of the momentum flux. Equation 3 assumes waves to be periodic, linear, and sinusoidal, and it ignores the momentum flux above the still-water level. However, in shallow water, waves are nonlinear with peaked crests and shallow troughs. The wave forces from these nonlinear waves can be very different from those estimated from linear waves. Equation 3 will underpredict the momentum flux under nonlinear wave crests. For  $M_F/\rho_w g h^2 = 0.2$ , Equation 3 will produce an underprediction error of roughly 10 percent while for  $M_F/\rho_w g h^2 = 0.8$ , the error will be roughly 100 percent.

The maximum wave momentum flux increases rapidly for nonlinear waves - steep waves in shallow water. This corresponds to the case where armor stability is at its minimum. It is desirable to develop a relation that can characterize the stability over the full range of water depths expected. Hughes (2003a, 2003b, 2004, 2005) estimated the nonlinear wave momentum flux using a numerical Fourier solution. The results were found to be well represented by the empirical equation:

$$\left( \frac{M_F}{\rho_w g h^2} \right)_{\max} = A_0 \left( \frac{h}{g T^2} \right)^{-A_1} \quad (4)$$

with

$$A_0 = 0.639 \left( \frac{H}{h} \right)^{2.026} \quad (5)$$

$$A_1 = 0.180 \left( \frac{H}{h} \right)^{-0.391}$$

Use of the nonlinear approximation in Equation 4 is important because stability is at its minimum when the incident wave is the most nonlinear. The actual wave momentum flux force that a particular armor unit is exposed to will vary from the value given by Equation 4 due to the slight error in the numerical approximation, the effects of the sloping foreshore, armor unit position in the water column, and armor unit extent of the water column. Therefore, a stability relation derived

using Equations 3 or 4 will be empirical. Herein, simply assume that the force on an armor unit is proportional to the depth-integrated maximum wave momentum flux given by Equation 4 computed at the structure toe.

**NEW ARMOR STABILITY EQUATION:** Generalized empirical stability equations were developed that characterize armor instability. These stability equations were fit to small-scale laboratory data from Van der Meer (1987). The data correspond to limited conditions as follows: mostly nonbreaking waves, normally-incident waves, nearshore slope of 1V:20H, mostly non-overtopped structures, narrow and wide armor gradations, permeable and impermeable structures, homogeneous structures, and angular randomly-placed armor stone. A small number of tests included shallow-water breaking waves and an additional small number included overtopped structures.

Two new equations resulted from the fit to data representing the two most important breaker types.

*Plunging waves*

$$N_m = 5.0(S/N_z^{0.5})^{0.2} P^{0.18} \sqrt{\cot \theta} \quad s_m \geq s_{mc} \quad (6)$$

*Surging waves*

$$N_m = 5.0(S/N_z^{0.5})^{0.2} P^{0.18} (\cot \theta)^{0.5-P} s_m^{-P/3} \quad s_m < s_{mc} \quad (7)$$

where

$$s_{mc} = -0.0035 \cot \theta + 0.028 \quad (8)$$

and

$$N_m = \left\{ \frac{K_a [(M_F)_{\max} / \gamma_w h^2]}{(S_r - 1)} \right\}^{1/2} \frac{h}{D_{n50}} \quad (9)$$

with  $K_a = 1$ . Here  $s_{mc}$  is the critical wave steepness on the structure,  $P$  = structure permeability,  $S = A_e/D_{n50}^2$  = normalized eroded area,  $A_e$  = eroded cross-sectional area,  $\gamma_w$  is the water specific weight,  $N_z = t/T_m$  = number of waves at mean period during event of duration  $t$ ,  $T_m$  = mean wave period,  $s_m = H_s/L_m$  = wave steepness,  $H_s$  significant wave height,  $L_m$  wavelength based on mean wave period, and  $\theta$  is the seaward structure slope from horizontal.

These stability equations are similar to those proposed by Hudson and Van der Meer. The greatest difference is the inclusion of a maximum momentum-flux-based wave force derived from first principles. The maximum wave force based on wave momentum flux should be reasonably well predicted, even in shallow water. Previous stability relations had no clear dependence on water depth. The new stability relations illustrate the well known fact that armor- stone stability decreases for increasing wave steepness in shallow water and is at a minimum for severe plunging breakers.

For plunging breakers, stability is only mildly a function of permeability and is not dependent on wave steepness. For surging waves, stability is more strongly related to permeability and wave steepness.

These equations are intended for preliminary design. It is recommended to utilize physical models, if at all possible, to verify stability for final design. Local experience is valuable, but details of wave focusing and instability at structure transitions will only be revealed in a physical model study.

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## EXAMPLE: COMPUTE STABLE ARMOR WEIGHT

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**Find:** The maximum wave momentum flux parameter and corresponding stable armor weight for a conventional nonovertopped rubble-mound breakwater with the following characteristics.

**Given:**

$\gamma_w$ = 64 pcf	– water specific weight
$S_r$ = 2.6	– stone specific gravity
$g$ = 32.2 ft/s <sup>2</sup>	– acceleration of gravity
$H_s$ = 7 ft	– significant wave height at structure toe
$T_m$ = 12 sec	– mean wave period
$h$ = 14 ft	– local water depth at structure toe
$S$ = 2	– start of damage
$N_z$ = 3,000	– storm duration = 6 hr at the peak of the storm
$P$ = 0.4	– structure permeability for traditional multilayer cross section
$\cot \theta$ = 2	– seaward structure slope of 1V:2H

Compute the maximum wave momentum flux at the structure toe using Equation 4:

$$A_0 = 0.639 \left( \frac{7}{14} \right)^{2.026} = 0.1569$$

$$A_l = 0.180 \left( \frac{7}{14} \right)^{-0.391} = 0.2360$$

$$\left( \frac{M_F}{\gamma_w h^2} \right)_{\max} = 0.6172$$

$$(M_F)_{\max} = 64(14)^2(0.1569) \left( \frac{14}{32.2(12)^2} \right)^{-0.236} = 7742.4$$

Compute local steepness of mean wavelength:  $s_m = H_s/L_m$  where  $L_m = 250$  ft is the local mean wavelength found using the linear wave dispersion relation with  $T_m = 12$  sec and  $h = 14$  ft. Therefore,  $s_m = 7/250 = 0.0280$ .

Compute critical wave steepness using Equation 8:  $s_{mc} = -0.0035(2) + 0.028 = 0.021$ .

In this case, the waves are plunging on the structure because  $s_m > s_{mc}$ . Equations 6 and 9, therefore, should be used to size the stone. Equation 6 for plunging waves on the structure, yields:

$$N_m = 5.0(2/3000^{0.5})^{0.2} 0.4^{0.18} \sqrt{2} = 3.093$$

Then the stable armor-stone size is given by Equation 9 as

$$D_n = \left[ \frac{1(0.6172)}{(2.6-1)} \right]^{1/2} \frac{14}{3.093} = 2.81 \text{ ft}$$

The corresponding armor weight is  $W = D_n^3 S_r \gamma_w = 2.81^3 (2.6) 64 = 3,698$  lb.

If all conditions are the same but the water depth is 28 ft, then the calculations would proceed as follows:

$$A_0 = 0.639 \left( \frac{7}{28} \right)^{2.026} = 0.0385$$

$$A_l = 0.180 \left( \frac{7}{28} \right)^{-0.391} = 0.3095$$

$$(M_F)_{\max} = 64(28)^2(0.0385) \left( \frac{28}{32.2(12)^2} \right)^{-0.3095} = 9398$$

$$\left( \frac{M_F}{\gamma_w h^2} \right)_{\max} = 0.1873$$

Compute local steepness of mean wavelength:  $s_m = H_s/L_m$  where  $L_m = 346$  ft is the local mean wavelength found using the linear wave dispersion relation with  $T_m = 12$  sec and  $h = 28$  ft. Therefore,  $s_m = 7/346 = 0.0202$ .

Compute critical wave steepness using Equation 8:  $s_{mc} = -0.0035(2) + 0.028 = 0.021$ .

In this case, the waves are surging on the structure because  $s_m < s_{mc}$ . Equations 7 and 9, therefore, should be used to size the stone. Equation 7 for surging waves on the structure, yields:

$$N_m = 5.0(2/3000^{0.5})^{0.2} 0.4^{0.18} \sqrt{2} (0.0202)^{-4/3} = 5.202$$

Then the stable stone size is given by Equation 9 as

$$D_n = \left[ \frac{1(0.1873)}{(2.6-1)} \right]^{1/2} \frac{28}{5.202} = 1.84 \text{ ft}$$

The corresponding armor weight is given by  $W = D_n^3 S_r \gamma_w = 1.84^3 (2.6) 64 = 1,039$  lb. Comparing the stone weights, the shallow water depth resulted in an increase in the stable stone weight by a factor of more than 3.5. The majority of the impact of shallower depth came from the momentum flux parameter in the stability number calculation.

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**POINTS OF CONTACT:** This CHETN was developed within the Risk Analysis of Coastal Structures work unit in the Navigation Systems R&D Program. The program is administered by Mr. James Clausner of the Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center. Questions about this Technical Note can be addressed to Dr. Jeffrey A. Melby ([Jeffrey.A.Melby@erdc.usace.army.mil](mailto:Jeffrey.A.Melby@erdc.usace.army.mil)) or Dr. Steven Hughes ([Steven.A.Hughes@erdc.usace.army.mil](mailto:Steven.A.Hughes@erdc.usace.army.mil)). Questions about the R&D program should be addressed to Mr. Clausner ([James.E.Clausner@erdc.usace.army.mil](mailto:James.E.Clausner@erdc.usace.army.mil)).

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